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A Naturally Renormalized Quantum Field Theory

S. Rouhani · M.V. Takook

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Abstract It was shown that quantum metric fluctuations smear out the singularities of Green functions on the light cone (Ford, arXiv:gr-qc/9707062), but it does not remove other ultraviolet divergences of the quantum field theory (QFT). We have proved that quantization in indefinite metric, *i.e.* QFT in Krein space, removes all divergences of the theory except light cone singularity (Gazeau, et al., Class. Quantum Gravity, 17:1415, 2000, arXiv:gr-qc/9904023; Takook, Int. J. Mod. Phys. E, 11:509, 2002, arXiv:gr-qc/0006019). In this paper, by considering the QFT in Krein space and the quantum metric fluctuations, it is shown that all divergences can be removed.

Keywords Quantum field · Krein space · Metric fluctuation

1 Introduction

Nowadays, one of the great challenges of physics is the achievement of a proper theory of quantized gravitational fields. In other words a theory which quantizes the gravitational fields without any anomaly. Such theory have been sought for the past seven decades without a thorough success. The causal structure of the space-time is one of the problems of quantum gravity. By introducing background field method this problem has been resolved although it could not be applied to the very early moments of evaluation of the universe (the Planck scale) where the perturbation of metric is in the same order as the background metric itself. The next problem which appears in the background field method is the non-renormaliziability of quantum gravity. Generally, there are three opinions about this problem. The first view sees the problem as an inherent problem of general relativity and the second, as an intrinsic problem of the quantum mechanics. The third view is that both,

S. Rouhani
Plasma Physics Research Center, Islamic Azad University, Tehran, Iran

M.V. Takook (✉)
Department of Physics, Razi University, Kermanshah, Iran
e-mail: takook@razi.ac.ir

quantum mechanics and general relativity are problematic and an alternative theory must be replaced.

Regularization and renormalization procedures have been successfully utilized to remove divergences in QFT. These procedures, however, cannot be extended to quantum gravity, and the theory remains divergent.

The singular behavior of Green function at short relative distances (ultraviolet divergence) or in the large relative distances (infrared divergence) leads to divergences in the QFT. The ultraviolet divergence appears in the following terms of Green function, $G(x,x')$, in the limit $x \to x'$ or $\sigma \to 0$:

$$\frac{1}{\sigma}, \quad \ln\sigma, \quad \text{and} \quad \delta(\sigma),$$

where σ is one-half of the square of the geodesic distance between x and x' ($\sigma = \frac{1}{2}(x-x')^2$). It was conjectured that quantum metric fluctuations might smear out the singularities of Green functions on the light cone *i.e.* $\delta(\sigma)$, but it does not remove other ultraviolet divergences of quantum field theory [1, 4, 5]. However, we showed quantization in Krein space removes all ultraviolet divergences of QFT except the light cone singularity [3]. In this procedure, the auxiliary negative norm states (negative frequency solution which do not interact with the physical states or real physical world) have been utilized. This method has been utilized for the covariant quantization of the minimally coupled scalar field in de Sitter space [2]. Indefinite metric quantization has been used in resolving other problems previously [6, 7]. Furthermore, we showed that the negative norm states can be an automatic renormalization device for certain problems [2, 3, 8–12].

Here, by considering the quantum field theory in Krein space and quantum metric fluctuations, we show that all divergences in quantum field theory are removed. We explicitly calculate the transition amplitude of the state $|q_1, q_2;\ \text{in}\rangle$ to the state $|p_1, p_2;\ \text{out}\rangle$ in s-channel contribution for $\lambda\phi^4$ theory, to one-loop approximation.

2 Krein Space Quantization

Recently, the existence of a non-zero cosmological constant has been proposed to explain the luminosity observations on the farthest supernovas [13, 14]. If this hypothesis is validated, our ideas on the large-scale universe should be changed and de Sitter (dS) metric will play an important role. Thus the quantization of the massless tensor spin-2 field in dS space *i.e.* a linear gravitational field, without infrared divergence presents an excellent modality for further researches. The linear quantum gravity is an important element in understanding quantum cosmology and quantum gravity. However, the graviton propagator in the linear approximation, for largely separated points, either has a pathological behavior (infrared divergence) or violates dS invariance [15–17]. Antoniadis, Iliopoulos and Tomaras [18] have shown that the pathological large-distance behavior of the graviton propagator in dS background does not manifest itself in the quadratic part of the effective action to the one-loop approximation. This means that the pathological behavior of the graviton propagator is gauge dependent and so should not appear in an effective way as a physical quantity. de Vega et al. [19] have also shown that the infrared divergence does not appear in the physical world. This result has been also obtained by other authors [20–23]. Linear gravity could indeed be constructed from a minimally coupled scalar field in dS [24, 25], and ambient space [26–29].[1]

[1] dS space is conveniently described as a hyperboloid embedded in a five-dimensional Minkowski space or ambient space.

It has been shown that for the minimally coupled scalar field in dS space, one can not construct a covariant quantization with only positive norm states. In addition, there appears an infrared divergence in the two-point function [24]. It has been proved that using two sets of solutions (positive and negative norm states) will be an unavoidable feature if one insists on preserving causality (locality), covariance and the elimination of the infrared divergence in QFT for the minimally coupled scalar field in dS space [2]. In other words we maintain the covariance principle and remove the positivity condition similar to the Gupta-Bleuler quantization of electrodynamics in Minkowski space.

As proved by Allen [24], the procedure of the covariant canonical quantization of the minimally coupled scalar field with positive norm states fails in dS space. Allen's result can be reformulated in the following way: the Hilbert space generated by a set of modes (named here the positive modes, including the zero mode) is not dS invariant,

$$\mathcal{H} = \left\{ \sum_{k\geq 0} \alpha_k \phi_k;\ \sum_{k\geq 0} |\alpha_k|^2 < \infty \right\}.$$

This means that it is not closed under the action of the generators of the dS group. In order to resolved this problem, we have to deal with an orthogonal set of positive and negative norm states, which is closed under an indecomposable representation of the dS group. The negative values of the inner product are precisely produced by the conjugate modes: $\langle \phi_k^*, \phi_k^* \rangle = -1$, $k \geq 0$.

We do insist on the fact that the space of solutions should contain "unphysical" states with negative norm. We use such states as a mathematical equipment in the quantization in Krein space. These states cannot propagate in the physical world and they only play the role of an automatic renormalization device in the theory. This auxiliary negative norm states are eliminated by imposing conditions on the field operator and physical states. One of the interesting results of this construction is that Green function, at large relative distances, does not diverge. In other words the previous infrared divergence disappears [2, 11] and the ultraviolet divergence in the stress tensor disappears as well, which means the quantum free scalar field in this method is automatically renormalized. By the use of this method for linear gravity (the traceless rank-2 "massless" tensor field) a fully covariant quantization in dS space is obtained [29] and the corresponding two-point function is free of any infrared divergence [10, 26].

Therefore, Quantum field operator in Krein space is defined by [2]

$$\phi(x) = \frac{1}{\sqrt{2}} \left[\phi_p(x) + \phi_n(x) \right], \tag{1}$$

where

$$\phi_p(x) = \sum_{k\geq 0} a_k \phi_k(x) + H.C., \qquad \phi_n(x) = \sum_{k\geq 0} b_k \phi^*(x) + H.C. \tag{2}$$

The positive mode $\phi_p(x)$ is the scalar field as was used by Allen. $a(\vec{k})$ and $b(\vec{k})$ are defined as two independent operators. The main departure from the standard QFT which is based on the CCR, lies in the requirement of the following commutation relations:

$$\begin{aligned} a_k|0\rangle &= 0, \qquad [a_k, a_{k'}^\dagger] = \delta_{kk'}, \\ b_k|0\rangle &= 0, \qquad [b_k, b_{k'}^\dagger] = -\delta_{kk'}. \end{aligned} \tag{3}$$

It is easy to show that a direct consequence of these formulas is the positivity of the energy *i.e.*

$$\langle\vec{k}|T_{00}|\vec{k}\rangle \geq 0,$$

where $|\vec{k}\rangle$ is a physical state (which is built from repeated action of $a_k^\dagger$'s on the vacuum). This quantity vanishes if and only if $|\vec{k}\rangle = |0\rangle$. Therefore, the normal ordering procedure for eliminating the ultraviolet divergence in the vacuum energy, which appears in the QFT (QFT in the Hilbert space) is not needed [2]. Another consequence of this formula is a covariant two-point function which is free of any infrared divergence [11].

Let us consider the quantization of the massive scalar field $\phi(x)$ in 4-dimensional Minkowski space-time. The relations (2) become

$$\phi_p(x) = \int d^3\vec{k}[a(\vec{k})u_p(k,x) + a^\dagger(\vec{k})u_p^*(k,x)],$$

$$\phi_n(x) = \int d^3\vec{k}[b(\vec{k})u_n(k,x) + b^\dagger(\vec{k})u_n^*(k,x)],$$

where $u_p = \frac{e^{-ik.x}}{\sqrt{(2\pi)^3 2k^0}}$ and $u_n = u_p^*$. Within the introduced framework, the "Wightman" two-point function is the imaginary part the usual Wightman two-point function which is built from the positive norm states

$$\mathcal{W}(x,x') = \langle 0|\phi(x)\phi(x')|0\rangle = \frac{1}{2}[\mathcal{W}_p(x,x') + \mathcal{W}_n(x,x')] = i\Im\mathcal{W}_p(x,x'), \tag{4}$$

where $\mathcal{W}_n = -\mathcal{W}_p^*$. The time-ordered product of fields is defined as

$$iG_T(x,x') = \langle 0|T\phi(x)\phi(x')|0\rangle = \theta(t-t')\mathcal{W}(x,x') + \theta(t'-t)\mathcal{W}(x',x). \tag{5}$$

In this case we obtain

$$G_T(x,x') = \frac{1}{2}[G_F^p(x,x') + (G_F^p(x,x'))^*] = \Re G_F^p(x,x'), \tag{6}$$

where the positive norm state time-ordered product of fields or Feynman two-point function is [30]

$$\begin{aligned} G_F^p(x,x') &= \int \frac{d^4k}{(2\pi)^4} e^{-ik.(x-x')}\tilde{G}^p(k) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik.(x-x')}}{k^2-m^2+i\epsilon} \\ &= -\frac{1}{8\pi}\delta(\sigma_0) + \frac{m^2}{8\pi}\theta(\sigma_0)\left[\frac{J_1(\sqrt{2m^2\sigma_0}) - iN_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}\right] \\ &\quad - \frac{im^2}{4\pi^2}\theta(-\sigma_0)\frac{K_1(\sqrt{2m^2(-\sigma_0)})}{\sqrt{2m^2(-\sigma_0)}}, \end{aligned} \tag{7}$$

where J_1, N_1 and K_1 are the Bessel functions. Then, the time-ordered product of fields in the Krein space becomes:

$$G_T(x,x') = \Re G_F^p(x,x') = \frac{-1}{8\pi}\delta(\sigma_0) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}, \quad x \neq x'. \tag{8}$$

Note that contribution of the coincident point singularity ($x = x'$) merely appears in the imaginary part of G_F ([11] and (9.52) in [31]). Equation (8) has only a singularity on the light cone. Quantum metric fluctuations can remove such singularity [1] which will be considered in the next section.

3 Quantum Metric Fluctuation

Consideration of a flat background space-time with a linearized perturbation $h_{\mu\nu}$ propagating upon it, constitute the basic modality of quantum metric fluctuation, *i.e.*

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h| < |\eta|. \tag{9}$$

In the unperturbed space-time, the square of the geodesic separation of points x and x' is defined by $2\sigma_0 = \eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu)$. In a general curved space and in the presence of the perturbation $h_{\mu\nu}$, it becomes 2σ where [31]

$$\sigma = \sigma_0 + \sigma_1 + O(h^2), \tag{10}$$

σ_1 is the first-order shift in σ (an operator in the linear quantum gravity). The impact of the metric fluctuation, in this semiclassical approach, on the second part of the Green function (8) is negligible. Its impact on the first term, however, could not be ignored and should be treated as follows.

In Minkowski space-time, the retarded Green function for a massless scalar field is

$$G^{(0)}_{ret}(x - x') = \frac{\theta(t - t')}{4\pi}\delta(\sigma_0), \tag{11}$$

which has a delta-function singularity on the future light cone and is zero elsewhere. In the presence of a classical metric perturbation, the retarded Green function has its delta-function singularity on the perturbed light cone, where $\sigma = 0$. In general, it may also become nonzero on the interior of the light cone due to backscattering off of the curvature. We are primarily interested in the behavior of functions near the perturbed light cone, so we can replace $G^{(0)}_{ret}(x - x')$ by

$$G_{ret}(x - x') = \frac{\theta(t - t')}{4\pi}\delta(\sigma). \tag{12}$$

This might be expressed as

$$G_{ret}(x - x') = \frac{\theta(t - t')}{8\pi^2}\int_{-\infty}^{\infty} d\alpha e^{i\alpha\sigma_0} e^{i\alpha\sigma_1}. \tag{13}$$

We now replace the classical metric perturbations by gravitons in a vacuum state $|\psi\rangle$. Then σ_1 becomes a quantum operator which is linear in the graviton field operator, $h_{\mu\nu}$. Because a squeezed vacuum state is a state that σ_1 may be decomposed into positive and negative frequency parts, *i.e.*, we may find σ_1^+ and σ_1^- so that $\sigma_1^+|\psi\rangle = 0$, $\langle\psi|\sigma_1^- = 0$, and $\sigma_1 = \sigma_1^+ + \sigma_1^-$. Thus when we average over the metric fluctuations, the retarded Green function is replaced by its quantum expectation value:

$$\langle G_{ret}(x - x')\rangle = \frac{\theta(t - t')}{8\pi^2}\int_{-\infty}^{\infty} d\alpha e^{i\alpha\sigma_0} e^{-\frac{1}{2}\alpha^2\langle\sigma_1^2\rangle}. \tag{14}$$

This integral converges only if $\langle\sigma_1^2\rangle > 0$, and the evaluation yields to

$$\langle G_{ret}(x-x')\rangle = \frac{\theta(t-t')}{8\pi^2}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}\exp\left(-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}\right). \tag{15}$$

Note that this averaged Green function is indeed finite at $\sigma_0 = 0$ provided that $\langle\sigma_1^2\rangle \neq 0$. Thus the light cone singularity has been removed [1].

According to the previous section, quantization in Krein space including the quantum metric fluctuation, removes all ultraviolet divergences of the theory:

$$\langle G_T(x-x')\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}\exp\left(-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}\right) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}. \tag{16}$$

In the case of $2\sigma_0 = \eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu) = 0$, due to the metric quantum fluctuation, $h_{\mu\nu}$, $\langle\sigma_1^2\rangle \neq 0$, and we have

$$\langle G_T(0)\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}} + \frac{m^2}{8\pi}\frac{1}{2}. \tag{17}$$

It should be noted that $\langle\sigma_1^2\rangle$ is related to the density of gravitons [1]. In this case the metric fluctuation is defined on the Minkowskian background and the masse m^2 can be defined respect to this background.

4 Interaction in Krein Space

In this section we study $\lambda\phi^4$ theory in flat space to one-loop approximation in the Krein space. In this case S-matrix elements which describe the scattering of the initial into the final physical states,

$$S_{fi} = \langle \text{out}, f|\text{in}, i\rangle = \langle \text{out}, f|S|\text{out}, i\rangle,$$

are the most important quantities to be calculated. The condition that the physical states must have positive norm leads one to write (see (1))

$$\begin{aligned} b_k|\text{phys. state}\rangle &= b_k|q_1,\ldots,q_m\rangle = 0,\\ \langle\text{phys. state}|b_k^\dagger &= \langle q_1,\ldots,q_m|b_k^\dagger = 0. \end{aligned} \tag{18}$$

This condition clearly indicates that the negative norm states do not appear in the external legs and naturally can not coupled with positive norm states in the external legs, *i.e.*

$$\langle\text{phys. state}|\text{unphys. state}\rangle = 0. \tag{19}$$

In fact, the initial physical states evolve to the sum of physical and unphysical states through the interaction. Also, the relations (18) and (19) indicate that negative norm states (or unphysical states) disappear in the external legs of S-matrix. In the internal legs, however, the propagation of negative norm states couple with the positive norm states therefore, we have a change in the Green function (16) and this directly leads to removing the divergences.

One can simply show that the LSZ reduction formula for the physical states can be written in terms of time-ordered product of the interaction field operators $\sqrt{2}\phi = \phi_p + \phi_n$ as the following [32],

$$\begin{aligned}&\langle p_1, \ldots, p_n;\ \text{out}|q_1, \ldots, q_m;\ \text{in}\rangle \\ &\quad = \text{disconnected graphs} + (iZ^{-1/2})^{(n+m)} \int d^4y_1 \cdots d^4x_m \\ &\quad\quad \times \prod_{i=1}^{n}\prod_{j=1}^{m} e^{ip_i\cdot y_i} e^{-iq_j\cdot x_j} (\Box_{y_1} + m^2)\cdots(\Box_{x_m} + m^2)\langle 0|T\phi(y_1)\cdots\phi(x_m)|0\rangle. \qquad (20)\end{aligned}$$

Similar to the usual quantization by using time evolution operator, the time-ordered product of the interaction field operators can be written in terms of time-ordered product of the free field operators:

$$\langle 0|T\phi(y_1)\cdots\phi(x_m)|0\rangle = \frac{\langle 0|T\phi_0(y_1)\cdots\phi_0(x_m)e^{i\int d^4z\mathcal{L}_{int}(\phi_0(z))}|0\rangle}{\langle 0|Te^{i\int d^4z\mathcal{L}_{int}(\phi_0(z))}|0\rangle}. \qquad (21)$$

By applying the Wick's theorem to the above equation, it can be written in terms of all possible products of time-ordered product of fields G_T (all of which are convergent in the ultraviolet and infrared limit). An immediate consequence of this construction is a finite quantum field theory.

Note that the tree order S-matrix elements of the QFT in Krein space are the same with the usual QFT. In the loop approximation, however, QFT in Krein space results in finite solutions. To demonstrate this we explicitly calculate the transition amplitude of the state $|q_1, q_2;\ \text{in}\rangle$ to the state $|p_1, p_2;\ \text{out}\rangle$ for s-channel contribution to the one-loop approximation. This is given by [32]

$$\begin{aligned}\mathcal{T} &\equiv \langle p_1, p_2;\ \text{out}|q_1, q_2;\ \text{in}\rangle_s = \int d^4y_1 d^4y_2 d^4x_1 d^4x_2 e^{ip_1.y_1+ip_2.y_2-iq_1.x_1-iq_2.x_2} \\ &\quad \times (\Box_{y_1} + m^2)(\Box_{y_2} + m^2)(\Box_{x_1} + m^2)(\Box_{x_2} + m^2)\frac{(-i\lambda)^2}{2!}\int d^4z_1 d^4z_2 [iG_T(z_1 - z_2)]^2 \\ &\quad \times G_T(y_1 - z_2)G_T(y_2 - z_1)G_T(x_1 - z_2)G_T(x_2 - z_2).\end{aligned}$$

Comparing with the Hilbert space quantization, we see that the Feynman Green function G_F^p is replaced by time-ordered product of the Green function G_T. So, we can write

$$\begin{aligned}\mathcal{T} &= \frac{\lambda^2}{2}\int d^4z_1 d^4z_2 e^{i(p_1+p_2).z_1 - i(q_1+q_2).z_2}[G_T(z_1 - z_2)]^2 \\ &= \frac{\lambda^2}{2}(2\pi)^4\delta^4(p_1 + p_2 - q_1 - q_2) \\ &\quad \times \int d^4z e^{i(p_1+p_2).z}\left(-\frac{1}{8\pi}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}\exp\left(-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}\right) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}\right)^2, \qquad (22)\end{aligned}$$

where $2\sigma_0 = (z_1 - z_2)^2 = z^2$. The second integral for the space-like separated pair (z_1, z_2) is zero. The first and second integrals are Gaussian and a Fourier transform of a non singular function, respectively. Therefore, the transition amplitude in Krein space quantization becomes finite to the one-loop approximation.

5 Conclusion

QFT in the Hilbert space contains some singularities which are eliminated by the renormalization and regularization procedures, so we say it is a re-normalizable theory. However, these procedures have not been successful in the quantum gravity. In this paper we showed quantization in the Krein space considering the quantum metric fluctuations leads to a theory which does not contain any singularity. In our previous works, we used this mathematical procedure to obtain a correct result for the Casimir effect; and also by extension of this method to the linear gravity (traceless part) in dS space we obtained a covariant two-point function which is free of the pathological large-distance behavior. As a future work it may be possible to use this method to obtain a theory of quantum gravity in the background field method which is free of any divergence.

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